

Theories of Neutrino Masses and Mixings

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Abstract

Recent developments in our understanding of neutrino masses and their implications for physics beyond the standard model are reviewed.

I. INTRODUCTION

History of weak interaction physics has to a large extent been a history of our understanding of the properties of the elusive spin half particles called the neutrinos. Evidence for only lefthanded neutrinos being emitted in beta decay was the cornerstone of the successful V-A theory of weak interactions suggested by Sudarshan, Marshak, Feynman and Gell-Mann; evidence for the neutral current interactions in early seventies provided brilliant confirmation of the successful gauge unification of weak and electromagnetic interactions proposed by Glashow, Salam and Weinberg.

Today as we enter a new millenium, we again have evidence for a very important new property of the neutrinos i.e. they have mass and as a result like the quarks, they mix with each other and lead to the phenomenon of neutrino oscillation. This is contrary to the expectations based on the standard model as well as the old V-A theory (in fact one may recall that one way to make the V-A theory plausible was to use invariance of the weak Lagrangian under the so called γ_5 invariance of all fermions, a principle which was motivated by the assumption that neutrinos have zero mass). The simple fact that neutrino masses vanish in the standard model is proof that its nonzero mass is an indication of new physics at some higher scale (or shorter distances). Study of details of neutrino masses and mixings is therefore going to open up new vistas in our journey towards a deeper understanding of the properties of the weak interactions at very short distances. This no doubt will have profound implications for the nature of the final theory of the particles, forces and the universe.

We are of course far from a complete picture of the masses and mixings of the various neutrinos and cannot therefore have a full outline of the theory that explains them. However there exist enough information and indirect indications that constrain the masses and mixings among the neutrinos that we can see a narrowing of the possibilities for the theories.

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Many clever experiments now under way will soon clarify or rule out many of the allowed models. It will be one of the goals of this article to give a panoramic view of the most likely scenarios for new physics that explain what is now known about neutrino masses [1]. We hope to emphasize the several interesting ideas for understanding the small neutrino masses and discuss in general terms how they can lead to scenarios for neutrinos currently being discussed in order to understand the observations. These ideas have a very good chance of being part of the final theory of neutrino masses. We then touch briefly on some specific models that are based on the above general framework but attempt to provide an understanding of the detailed mass and mixing patterns. These works are instructive for several reasons: first they provide proof of the detailed workability of the general ideas described above (sort of existence proofs that things will work); second they often illustrate the kind of assumptions needed and through that a unique insight into which directions the next step should be; finally of course nature may be generous in picking one of those models as the final message bearer.

II. NEUTRINO MASS IS DIFFERENT FROM ELECTRON MASS

The electron and the neutrino are in many ways very similar particles: they are both spin half objects; they both participate in weak interactions with same strength; in fact they are so similar that in the limit of exact gauge symmetries they are two states of the same object and therefore in principle indistinguishable. Yet there are profound differences between them in the standard model: after gauge symmetry breaking, only electron has electric charge. Another difference is that only the lefthanded neutrino is included in the standard model and not its righthanded counterpart; this is unlike the electron both whose helicity states are included. In fact it is this property of the standard model coupled with $B - L$ being an exact symmetry which leads to neutrino remaining massless to all orders in perturbation theory as well as after the inclusion of nonperturbative effects.

The fact that the neutrino has no electric charge endows it with certain properties not shared by other fermions of the standard model such as the quarks and the electron (all of whom are electrically charged). The point is that for neutral fermions one can write two kinds of Lorentz invariant mass terms, the Dirac and Majorana masses, whereas for the charged fermions, conservation of electric charge allows only the Dirac type mass terms. In the four component notation for the fermions, the Dirac mass has the form $\bar{\psi}\psi$, whereas the Majorana mass is of the form $\psi^T C^{-1}\psi$, where ψ is the four component spinor and C is the charge conjugation matrix. One can also discuss the two different kinds of mass terms using the two component notation for the spinors. If we denote by χ and ϕ the two two-component spinors which make up the four component object ψ , then a Dirac mass is $\chi^T \sigma_2 \phi$ whereas a Majorana mass is given by $\chi^T \sigma_2 \chi$, where σ_a are the Pauli matrices. To make correspondence with the four component notation, we point out that χ and $i\sigma_2 \phi^*$ are nothing but the ψ_L and ψ_R respectively. It is then clear that χ and ϕ have opposite electric charges; therefore the Dirac mass $\chi^T \sigma_2 \phi$ maintains electric charge conservation (as well as any other kind of charge like lepton number etc.).

This richness in the possibility for neutrino masses also has a down side in the sense that in general, there are more parameters describing the masses of the neutrinos than those for the quarks and leptons. For instance for the electron and quarks, dynamics (electric charge

conservation) reduces the number of parameters in their mass matrix. As an example, using the two component notation for all fermions, for the case of two two component spinors, a charged fermion mass will be described only by one parameters whereas for a neutrino, there will be three parameters. This difference inceases rapidly e.g. for $2N$ spinors, to describe charged fermion masses, we need N^2 parameters (ignoring CP violation) whereas for neutrinos, we need $\frac{2N(2N+1)}{2}$ parameters. What is more interesting is that for a neutrino like particle, one can have both even and odd number of two component objects and have a consistent theory. There are suggestions that if there are an odd number of two component charged spinors, the corresponding theory is not consistent as a perturbative field theory.

In this article, we will use two component notation for neutrinos. Thus when we say that there are N neutrinos, we will mean N two-component neutrinos.

In the two component language, all massive neutrinos are Majorana particles and what is conventionally called a Dirac neutrino is really a very specific choice of mass parameters for the Majorana neutrino. Let us give some examples: If there is only one two component neutrino (we will drop the prefix two component henceforth), it can have a mass $m\nu^T\sigma_2\nu$ (to be called $\equiv m\nu\nu$ in shorthanded notation). The neutrino is now a self conjugate object which can be seen if we write an equivalent 4-component spinor ψ :

$$\psi = \begin{pmatrix} \nu \\ i\sigma_2\nu^* \end{pmatrix} \quad (1)$$

Note that this 4-component spinor satisfies the condition

$$\psi = \psi^c \equiv C\bar{\psi}^T \quad (2)$$

This condition implies that the neutrino is its own anti-particle, a fact more transparent in the 4- rather than the two-component notation. The above exercise illustrates an important point i.e. given any two component spinor, one can always write a self conjugate (or Majorana) 4-component spinor. Whether a particle is really its own antiparticle or not is therefore determined by its interactions. To see this for the electrons, one may solve the following excercise i.e. if we wrote two Majorana spinors using the two two component spinors that describe the charged fermion (electron), then until we turn on the electromagnetic interactions and the mass term, we will not know whether the electron is its own antiparticle or not. Once we turn on the electromagnetism, this ambiguity is resolved.

Let us now go one step further and consider two 2-component neutrinos (ν_1, ν_2). The general mass matrix for this case is given by:

$$\mathcal{M}_{2 \times 2} = \begin{pmatrix} m_1 & m_3 \\ m_3 & m_2 \end{pmatrix} \quad (3)$$

Note first that this is a symmetric matrix and can be diagonalized by orthogonal transformations. The eigenstates which will be certain admixtures of the original neutrinos now describe self conjugate particles. One can look at some special cases:

Case i:

If we have $m_{1,2} = 0$ and $m_3 \neq 0$, then one can assign a charge $+1$ to ν_1 and -1 to ν_2 and the theory has an extra $U(1)$ symmetry which can be identified as the lepton number and the particle is then called a Dirac neutrino. The point to be noted is that the Dirac neutrino

is a special case of for two Majorana neutrinos. In fact if we insisted on calling this case one with two Majorana neutrinos, then the two will have equal and opposite (in sign) mass as can be seen diagonalizing the above mass matrix. Thus a Dirac neutrino can be thought of as two Majorana neutrinos with equal and opposite (in sign) masses. Since the argument of a complex mass term in general refers to its C transformation property (i.e. $\psi^c = e^{i\delta_m}\psi$, where δ_m is the phase of the complex mass term), the two two component fields of a Dirac neutrino have opposite charge conjugation properties.

Case ii:

If we have $m_{1,2} \ll m_3$, this case is called pseudo-Dirac neutrino since this is a slight departure from case (i). In reality, in this case also the neutrinos are Majorana neutrinos with their masses $\pm m_0 + \delta$ with $\delta \ll m_0$. The two component neutrinos will be maximally mixed. Thus this case is of great current physical interest in view of the atmospheric (and perhaps solar) neutrino data.

Case iii:

There is third case where one may have $m_1 = 0$ and $m_3 \ll m_2$. In this case the eigenvalues of the neutrino mass matrix are given respectively by: $m_\nu \simeq -\frac{m_3^2}{m_2}$ and $M \simeq m_2$. One may wonder under what conditions such a situation may arise in a realistic gauge model. It turns out that if ν_1 transforms as an $SU(2)_L$ doublet and ν_2 is an $SU(2)_L$ singlet, then the value of m_3 is limited by the weak scale whereas m_2 has no such limit and $m_1 = 0$ if the theory has no $SU(2)_L$ triplet field (as for instance is the case in the standard model). Choosing $m_2 \gg m_3$ then provides a natural way to understand the smallness of the neutrino masses. This is known as the seesaw mechanism [2]. Since this case is very different from the case (i) and (ii), it is generally said that in grand unified theories, one expects the neutrinos to be Majorana particles. The reason is that in most grand unified theories there is a higher scale which under appropriate situations provides a natural home for the large mass m_2 .

While we have so far used only two neutrinos to exemplify the various cases including the seesaw mechanism, these discussions generalize when $m_{1,2,3}$ are each $N \times N$ matrices (which we denote by $M_{1,2,3}$). For example, the seesaw formula for this general situation can be written as

$$\mathcal{M}_\nu \simeq -M_{3D}^T M_{2R}^{-1} M_{3D} \quad (4)$$

where the subscripts D and R are used in anticipation of their origin in gauge theories where M_D turns out to be the Dirac matrix and M_R is the mass matrix of the right handed neutrinos and all eigenvalues of M_R are much larger than the elements of M_D . It is also worth pointing out that Eq. (4) can be written in a more general form where the Dirac matrices are not necessarily square matrices but $N \times M$ matrices with $N \neq M$. We give such examples below.

Although there is no experimental proof that the neutrino is a Majorana particle, the general opinion is that the seesaw mechanism provides such a simple way to understand the glaring differences between the masses of the neutrinos and the charged fermions that neutrinos indeed must be Majorana particles.

Even though for most situations, the neutrino can be treated as a two component object regardless of whether its mass is of Dirac or Majorana type, there are certain practical situations where differences between the Majorana and Dirac neutrino becomes explicit: one case is when the two neutrinos annihilate. For Dirac neutrinos, the particle and the

antiparticle are distinct and therefore their annihilation is not restricted by Pauli principle in any manner. However, for the case of Majorana neutrinos, the identity of neutrinos and antineutrinos plays an important role and one finds that the annihilation to the Z-bosons occurs only via the P-waves. Similarly in the decay of the neutrino to any final state, the decay rate for the Majorana neutrino is a factor of two higher than for the Dirac neutrino.

III. EXPERIMENTAL INDICATIONS FOR NEUTRINO MASSES

As has been extensively discussed elsewhere in this book, while the direct search experiments for neutrino masses using tritium beta decay and neutrinoless double beta decay have only yielded upper limits, the searches for neutrino oscillation, which can occur only if the neutrinos have masses and mixings have yielded positive evidences. There is now clear evidence from one and strong indications from other experiments for neutrino oscillations and hence neutrino masses. The evidence comes from the atmospheric neutrino data in the Super-Kamiokande experiment [3] which confirms the indications of oscillations in earlier data from the Kamiokande [4], IMB [5] experiments. More recent data from Soudan II [6] and MACRO [7] experiments provide further confirmation of this evidence. The observation here is the following: in the standard model with massless neutrinos, all the muon and electron neutrinos produced at the top of the atmosphere would be expected to reach detectors on the earth and would be isotropic; what has been observed is that while that is true for the electron neutrinos, the muon neutrino flux observed on earth exhibit a strong zenith angle dependence. A simple way to understand this would be to assume that the muon neutrinos oscillate into another undetected species of neutrino on their way to the earth, with a characteristic oscillation length of order of ten thousand kilometers. Since the oscillation length is roughly given by $E(\text{GeV})/\Delta m^2(\text{eV}^2)$ kilometers, for a GeV neutrino, one would expect the particle physics parameter Δm^2 corresponding to the mass difference between the two neutrinos to be around 10^{-3} eV^2 corresponding to maximal mixing.

From the existing data several important conclusions can be drawn: (i) the data cannot be fit assuming oscillation between ν_μ and ν_e ; (ii) two oscillation scenarios that fit the data are $\nu_\mu - \nu_\tau$ as well as $\nu_\mu - \nu_s$ oscillations (where ν_s is a sterile neutrino that does not couple to the W or the Z bosons in the basic Lagrangian), although at the two σ level, the first scenario is a better fit than the latter. The more precise values of the oscillation parameters at 90% c.l. are:

$$\begin{aligned} \Delta m_{\nu_\mu \nu_\tau}^2 &\simeq (2 - 8) \times 10^{-3} \text{ eV}^2; \\ \sin^2 2\theta_{\mu\tau} &\simeq 0.8 - 1 \end{aligned} \tag{5}$$

The second evidence for neutrino oscillation comes from the five experiments that have observed a deficit in the flux of neutrinos from the Sun as compared to the predictions of the standard solar model championed by Bahcall and his collaborators [8] and more recently studied by many groups. The experiments responsible for this discovery are the Chlorine, Kamiokande, Gallex, SAGE and Super-Kamiokande [9] experiments conducted at the Homestake mine, Kamioka in Japan, Gran Sasso in Italy and Baksan in Russia. The different experiments see different parts of the solar neutrino spectrum. The details of these considerations are discussed in other chapters. The oscillation interpretation of the solar

neutrino deficit has more facets to it than the atmospheric case: first the final state particle that the ν_e oscillates into and second what kind of Δm^2 and mixings fit the data. At the moment there is a multitude of possibilities. Let us summarize them now.

As far as the final state goes, it can either be one of the two remaining active neutrinos, ν_μ and ν_τ or it can be the sterile neutrino ν_s as in the case of atmospheric neutrinos. Both possibilities are open at the moment. The SNO experiment which is expected to measure the solar neutrino flux via neutral current interactions will settle the issue of whether the final state of solar neutrino oscillation is the active neutrino or the sterile neutrino; in the former case, the ratio r of charged current flux (Φ_{CC}) to the neutral current flux (Φ_{NC}) is nearly half whereas in the latter case, it is one. As far as the Δm^2 and $\sin^2 2\theta$ parameters go, there are three possibilities: if the oscillation proceeds without any help from the matter in the dense core of the Sun, it is called vacuum oscillation (VO); if oscillation is enhanced by the solar core, it is called the MSW mechanism, in which case there are two ranges of parameters that can explain the deficit- the small angle (SMA-MSW) and the large angle (LMA-MSW). The parameter ranges, taken from Ref. [8] are:

$$\begin{aligned} VO : \Delta m^2 &\simeq 6.5 \times 10^{-11} eV^2; \sin^2 2\theta \simeq 0.75 - 1 \\ SMA - MSW : \Delta m^2 &\simeq 5 \times 10^{-6} eV^2; \sin^2 2\theta \simeq 5 \times 10^{-3} \\ LMA - MSW : \Delta m^2 &\simeq 1.2 \times 10^{-5} - 3.1 \times 10^{-4} eV^2; \sin^2 2\theta \simeq 0.58 - 1.00 \end{aligned} \quad (6)$$

Relevant point to note is that there is no large angle MSW fit for the case of sterile neutrinos due to absence of matter effect for them. The situation lately however has been quite fluid in the sense that there are measurements from the Super-Kamiokande experiment of the electron energy distribution which appears to contradict the MSW-SMA solution; similarly there are day night effects which seem to prefer MSW-LMA solution although VO solutions also lead to day night effects due to matter effects for certain ranges of Δm^2 [10]. There are also indications of seasonal variation of the solar neutrino flux above and beyond that expected from the position of the earth in the orbit.

Finally, we come to the last indication of neutrino oscillation from the Los Alamos Liquid Scintillation Detector (LSND) experiment [11], where neutrino oscillations of ν_μ both from a stopped muon decay (DAR) as well as the one accompanying the muon in pion decay (DIF) have been observed. The evidence from the DAR is statistically more significant and is an oscillation from $\bar{\nu}_\mu$ to $\bar{\nu}_e$. The mass and mixing parameter range that fits data is:

$$LSND : \Delta m^2 \simeq 0.2 - 2 eV^2; \sin^2 2\theta \simeq 0.003 - 0.03 \quad (7)$$

There are also points at higher masses specifically at 6 eV² which are allowed by the present LSND data for small mixings [12]. Presently KARMEN experiment at the Rutherford laboratory is also searching for the same oscillation. While they have found about eight events as of this writing, this is consistent with their expected background [13]. The proposed MiniBooNE experiment Fermilab [12] will provide more definitive information on this very important process in the next five years.

Our goal now is to study the theoretical implications of these discoveries. We will proceed towards this goal in the following manner: we will isolate the mass patterns that fit the above data and then look for plausible models that can first lead to the general feature that neutrinos have tiny masses; then we would try to understand in simple manner some of

the features indicated by data in the hope that these general ideas will be part of our final understanding of the neutrino masses. As mentioned earlier on, to understand the neutrino masses one has to go beyond the standard model. First we will sharpen what we mean by this statement. Then we will present some ideas which may form the basic framework for constructing the detailed models. We will refrain from discussing any specific models except perhaps giving examples by way of illustrating the theoretical ideas.

IV. PATTERNS AND TEXTURES FOR NEUTRINOS

As already mentioned, we will assume two component neutrinos and therefore their masses will in general be Majorana type. Let us also give our notation to facilitate further discussion: the neutrinos emitted in weak processes such as the beta decay or muon decay are weak eigenstates and are not mass eigenstates. The latter determines how a neutrino state evolves in time. Similarly, in the detection process, it is the weak eigenstate that is picked out. This is of course the key idea behind neutrino oscillation [14]. It is therefore important to express the weak eigenstates in terms of the mass eigenstates. We will denote the weak eigenstate by the symbol α, β or simply e, μ, τ etc whereas the mass eigenstate will be denoted by the symbols i, j, k etc. The mixing angles will be denoted by $U_{\alpha i}$ and relate the two sets of eigenstates as follows:

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = U \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix} \quad (8)$$

Using this equation, one can derive the wellknown oscillation formulae for the survival probability of a particular weak eigenstate α to be:

$$P_{\alpha\alpha} = 1 - 4 \sum_{i < j} |U_{\alpha i}|^2 |U_{\alpha j}|^2 \sin^2 \Delta_{ij} \quad (9)$$

where $\Delta_{ij} = \frac{(m_i^2 - m_j^2)L}{4E}$. The transition probability from one weak eigenstate to another is given by

$$P_{\alpha\beta} = 4 \sum_{i < j} U_{\alpha i} U_{\beta i}^* U_{\alpha j}^* U_{\beta j} \sin^2 \Delta_{ij} \quad (10)$$

Since Majorana masses violate lepton number, a very important constraint on any discussion of neutrino mass patterns arises from the negative searches for neutrinoless double beta decay [15]. The most stringent present limits are obtained from the Heidelberg-Moscow enriched Germanium-76 experiment at Gran Sasso and implies an upper limit on the following combination of masses and mixings:

$$< m_\nu > \equiv \sum_i U_{ei}^2 m_{\nu_i} \leq 0.2 \text{ eV} \quad (11)$$

This upper limit depends on the nuclear matrix element calculated by the Heidelberg group [16]. There could be an uncertainty of a factor of two to three in this estimate. This would then relax the above upper bound to at most 0.6 eV. This is a very stringent limit and

becomes especially relevant when one considers whether the neutrinos constitute a significant fraction of the hot dark matter of the universe. A useful working formula is $\sum_i m_{\nu_i} \simeq 24\Omega_\nu$ eV where Ω_ν is the neutrino fraction that contributes to the dark matter of the universe. For instance, if the dark matter fraction is 20%, then the sum total of neutrino masses must be 4.8 eV. The situation at the moment is uncertain [18] after the results from the high z supernova searches indicated possible nonzero cosmological constant. Nevertheless, from structure formation data, a total neutrino mass of 2-3 eV cannot strictly be ruled out and in fact one particular fit [19] prefers a cold+hot dark matter with a similar mass as a better fit than any other (e.g. CDM+ Λ). The proposed GENIUS [17] experiment by the Heidelberg-Moscw collaboration has the promise to bring down the upper limit on $\langle m_\nu \rangle$ by two orders of magnitude. This would profoundly effect the current ideas on neutrino masses and will help to more sharply define the theoretical directions in the field.

In view of several levels of uncertainties that currently surround the various pieces of information on neutrino masses, we will consider different scenarios. It is not an unfair reflection of the present community consensus to say that the solar and the atmospheric neutrino results are the most secure evidences for neutrino masses. We will therefore first consider the implications of taking these two sets of data seriously supplemented by the very useful information from neutrinoless double beta decay. We will include the LSND data subsequently.

A. Solar and Atmospheric data and neutrino mass patterns

If one wants to fit only the solar and atmospheric neutrino data, regardless of the nature of the solution to the solar neutrino puzzle (i.e. MSW small or large angle or vacuum oscillation), it is possible to have consistent scenarios using only the three known neutrinos. There are many mixing patterns and neutrino mass matrices that can be used for the purpose. In discussing these patterns, it is important to remember that a solution to the atmospheric neutrino puzzle requires that in the context of a three neutrino model, ν_μ and ν_τ must mix maximally. There are two interesting mass patterns that have been widely discussed in literature: (i) hierarchical pattern with $m_1 \ll m_2 \ll m_3$ or (ii) approximately degenerate pattern [20] $m_1 \simeq m_2 \simeq m_3$, where m_i are the eigenvalues of the neutrino mass matrix. In the first case, the atmospheric and the solar neutrino data give direct information on m_3 and m_2 respectively. On the other hand, in the second case, the mass differences between the first and the second eigenvalues are chosen to fit the solar neutrino data and the second and the third to fit the atmospheric neutrino data. Neutrinoless double beta decay limits imply very stringent constraints on the mixing pattern in the degenerate case; but before we proceed to this discussion let us focus for a while on the hierarchical mass pattern.

In proceeding with this discussion it is important to remember that the CHOOZ reactor data [21] implies that for $\Delta m^2 \geq 10^{-3}$ eV², the electron neutrino mixing angle has a rough upper bound $|U_{ei}| \leq 0.2$. Furthermore it is now certain that atmospheric neutrino data cannot be fitted by $\nu_\mu - \nu_e$ oscillation. Together they imply that in any neutrino mass matrix construction, one must require that $|U_{e3}| \leq 0.2$ [22]. Note that U_{e2} can be larger since the relevant Δm^2 which corresponds to the solar neutrino puzzle is lower than what the CHOOZ experiment is sensitive to. On the other hand the solar neutrino admits both

small and large mixing angles. Combining these two inputs, one can conclude that the 3×3 neutrino mixing matrix is given by [23]

$$U_\nu = \begin{pmatrix} c & s & 0 \\ \frac{s}{\sqrt{2}} & \frac{c}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ \frac{s}{\sqrt{2}} & \frac{c}{\sqrt{2}} & +\frac{1}{\sqrt{2}} \end{pmatrix} \quad (12)$$

In writing the above mixing matrix, we have assumed atmospheric neutrino data fit with $\sin^2 2\theta = 1$. On the other hand if we took, this value to be $\sin^2 2\theta = 8/9$, the above mixing matrix changes to:

$$U_\nu = \begin{pmatrix} c & s & 0 \\ \frac{s}{\sqrt{3}} & \frac{c}{\sqrt{3}} & -\frac{\sqrt{2}}{\sqrt{3}} \\ \frac{s\sqrt{2}}{\sqrt{3}} & \frac{c\sqrt{2}}{\sqrt{3}} & +\frac{1}{\sqrt{3}} \end{pmatrix} \quad (13)$$

There are two special cases, where these mixing matrices take specially appealing forms: case (A): if maximal mixing solutions are chosen for the solar neutrino puzzle or case(B): it is assumed that the neutrino masses are degenerate with a mass m_0 such that the effective mass deduced from neutrinoless double beta decay $< m_\nu > \ll m_0$ [24]. In either case, the first matrix reduces to the so called bimaximal form [23–25] whereas the second matrix reduces to the democratic form [26]. The mixing matrices are obtained from the above two equations by taking $c = s = \frac{1}{\sqrt{2}}$. The neutrino mass matrices in both these cases can be obtained by writing

$$\mathcal{M}_\nu = U M^d U^T \quad (14)$$

where $M^d = \text{Diag}(m_1, m_2, m_3)$.

Degenerate case:

The most compelling physical motivation for this case comes from the requirement that neutrinos constitute a significant fraction of the dark matter of the universe. Using our previous discussion, we see that if the neutrino HDM constitutes about 20% of the critical mass of the universe, then the total neutrino mass i.e. $\sum_i |m_i|$ must be about 4.8 eV. If all three neutrinos are degenerate the share of each species is 1.6 eV. Note that this is much bigger than the present upper limits on the effective mass $< m_\nu >$ from neutrinoless double beta decay. If we ignore CP phases, then this implies a strong constraint on the mixings.

Two particularly interesting mass matrices emerge, for the case where the neutrino masses are approximately degenerate. To derive them, let us choose $M^d = \text{Diag}(m_0, -m_0, m_0)$. If we further assume that $U_{e3} \simeq 0$, then we conclude that the mixing matrix elements $U_{e1} \simeq U_{e2} \simeq \frac{1}{\sqrt{2}}$. It therefore follows that the small angle MSW solution is automatically eliminated. For the bimaximal case, one gets the neutrino mass matrix of the form [27]:

$$M_\nu = m_0 \begin{pmatrix} 0 & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{2} & -\frac{1}{2} \\ \frac{1}{\sqrt{2}} & -\frac{1}{2} & \frac{1}{2} \end{pmatrix} \quad (15)$$

For derivation of this mass matrix in gauge models, see Ref. [28].

There is a corresponding mass matrix for the democratic case: it is given by

$$M_\nu = m_0 \begin{pmatrix} 0 & \frac{1}{\sqrt{3}} & \frac{\sqrt{2}}{\sqrt{3}} \\ \frac{1}{\sqrt{3}} & \frac{2}{3} & -\frac{\sqrt{2}}{3} \\ \frac{\sqrt{2}}{\sqrt{3}} & -\frac{\sqrt{2}}{3} & \frac{1}{3} \end{pmatrix} \quad (16)$$

Finally we want to note the form of the neutrino mass matrices that lead in general case to mixing matrices which have either the democratic or the bimaximal form regardless of the nature of the eigenvalues. The importance of this discussion is that in trying to construct gauge models for neutrinos, the mass matrix follows directly from the Lagrangian and the mixing follows from this afterwards.

(i) *Bimaximal case*

$$M_\nu = \begin{pmatrix} A + D & F & F \\ F & A & D \\ F & D & A \end{pmatrix} \quad (17)$$

Note that vanishing of neutrinoless double beta decay implies that $A = -D$. A special case of this is when $A = D = 0$. Such mass matrices have been discussed in literature for the three neutrino case in Ref. [30] and for construction of gauge models for this case, see [31].

(ii) *Democratic case:*

This case is more complicated. As was noted in Ref. [29] one must choose the charged lepton mass matrix in the form while keeping the neutrino mass matrix diagonal.:

$$M_\ell = \begin{pmatrix} a & 1 & 1 \\ 1 & a & 1 \\ 1 & 1 & a \end{pmatrix} \quad (18)$$

The symmetric form of the matrix in the democratic case is clear and in fact it exhibits a permutation symmetry S_3 operating on the lepton doublets, which provides another clue to possible gauge model building.

Hierarchical case:

One can also get some clue to model building by analysing the case where the neutrino masses are hierarchical i.e. $m_1 \ll m_2 \ll m_3$. To see this let us first ignore the couplings of the first generation and consider only the 2×2 mass matrix involving the second and the third generation. It has been pointed out that [32], one very simple way to get a natural hierarchy while having a maximal (or large) mixing is to have a matrix M_ν of rank one i.e.

$$M_\nu = \begin{pmatrix} x^2 & x \\ x & 1 \end{pmatrix} \quad (19)$$

This matrix has one zero eigenvalue whereas choosing $x \simeq 1$ leads to large mixing angle. Further interest in this idea emerges from the observation that such an M_ν can arise via the

seesaw mechanism if one chooses $M_D = \begin{pmatrix} 0 & 0 \\ x & 1 \end{pmatrix}$. The interesting point about this form for M_D is that if we define it as $\bar{\nu}_L M_D \nu_R$ (with $\nu_{L,R}$ are column vectors), the above form for M_D only mixes the right handed neutrinos and not the left handed ones. In a quark lepton unified model this would mean that only right handed quarks mix with a large mixing angle. This of course is completely unconstrained by observations that confirm the standard model since there are no righthanded charged currents in the standard model. More importantly, it leaves the left handed mixing small. Thus this observation may provide one reconciliation of the apparent conundrum that the quark mixing angles are small whereas the neutrino mixing angles are large. This particular form has the disadvantage that it cannot arise in models such as left-right models which generally lead to symmetric Dirac masses.

There are also other ways to generate maximal mixing angles. For instance a choice for neutrino mass matrix (for the 2-3 sector) of the form

$$M_{23} = \begin{pmatrix} a & b \\ b & a \end{pmatrix} \quad (20)$$

leads to maximal mixing. These mass matrices respect permutation symmetries (e.g. S_3) and therefore can be stable under radiative corrections. Ref. [33] has discussed the stability of neutrino mass matrices with respect to small variations of the parameters for various neutrino mass hierarchies.

B. Solar, atmospheric and LSND data and scenarios with sterile neutrinos

In order to explain solar, atmospheric and the LSND data simultaneously using the oscillation picture, one must invoke additional neutrinos since the different Δm^2 s needed to explain each piece of data is very different from the others and would never add up to zero as would be the case if there were only three neutrino species i.e. $\Delta m_{12}^2 + \Delta m_{23}^2 + \Delta m_{31}^2 = 0$. This is a revolutionary result since LEP and SLC measurements imply precisely (within very small errors) that there are almost precisely three active neutrinos coupling to the Z-boson (the actual LEP result is $N_\nu = 2.994 \pm 0.012$ [34]). Furthermore, even though the conclusions from big bang nucleosynthesis [35] are not as precise as the LEP-SLC data, the Helium abundance is very sensitive to the number of neutrinos and anywhere from 3.2 to 4.4 total number of neutrinos could be accommodated. Already these two results imply strong constraints on the theories that include extra neutrinos beyond those present in the standard model. For instance, the LEP-SLC data implies that the extra neutrinos cannot couple to the Z-boson with the same strength as other neutrinos. They have therefore been designated in the literature as sterile neutrinos. Their “sterility” also helps them to evade the bounds from big bang nucleosynthesis as follows. Suppose, the coupling of a sterile neutrino to known leptons is given by the four Fermi interaction with a strength given by $G_F \epsilon$. Then, if $\epsilon \leq 10^{-3}$ [1], then the sterile neutrinos decouple before the QCD phase transition temperature of about a 100 MeV and their effective contribution at the BBN era becomes equivalent to about 0.1 neutrino species. There are further constraints on Δm^2 and $\sin^2 2\theta$ that arise when the sterile neutrinos oscillate at the BBN era [36]. Typically, the constraint is

$$\Delta m_{es}^2 \sin^2 2\theta_{es} \leq 10^{-6} \text{eV}^2 \quad (21)$$

These considerations are important in constructing detailed theories of the sterile neutrinos. While we postpone the detailed discussion of theories with extra neutrinos to a separate section, here we wish to study the possible four or more neutrino patterns and their mass matrices that correspond to the existing neutrino data.

B1. Four neutrino case

It is possible to construct three possible scenarios for neutrino data using the four neutrinos. Denoting the sterile neutrino by ν_s , the three scenarios are the following:

Case B1.1:

The solar neutrino data is solved via the MSW small angle oscillation between the $\nu_e - \nu_s$, which then have a mass difference of $\Delta m_{es}^2 \simeq 10^{-5} \text{eV}^2$ whereas the atmospheric neutrino data is solved via the $\nu_\mu - \nu_\tau$ oscillation [20,37]. (Of course, the solar neutrino puzzle could also be solved via the vacuum oscillation of $\nu_e - \nu_s$.) In either case, if we have $m_{\nu_\mu} \simeq m_{\nu_\tau} \simeq \text{eV}$, then there can be a significant hot dark matter component in the universe with enormous implications for structure formation. Note that LSND data is then fitted by $\nu_\mu - \nu_e$ oscillation with the m_{ν_μ} being determined by Δm_{LSND}^2 . It is interesting that the present LSND data has a considerable range where the neutrinos contribute significantly to the HDM component of the universe.

Case B1.2:

It is also possible to have a scenario where the atmospheric neutrino data is fitted by the $\nu_\mu - \nu_s$ maximal oscillation whereas the solar neutrino data is fitted by the $\nu_e - \nu_\tau$ oscillation. Again as before, the Δm_{LSND}^2 determines the splitting between the two pairs of levels and will determine the hot dark matter content of the universe. In a series of papers, Bilenky et al have extensively studied the tests of these models [38].

It is possible to experimentally distinguish between these two scenarios. The $\nu_e - \nu_s$ oscillation solution to the solar neutrino puzzle can be tested once SNO experiment measures the neutral current flux of the solar neutrinos since the ν_s 's do not have any neutral current interaction as opposed to the $\nu_{\mu,\tau}$. Similarly, in the atmospheric neutrino data search for neutral current events with pion production $\nu + N \rightarrow N + \pi^0 + N$ can discriminate between the oscillation of the atmospheric ν_μ 's to ν_τ 's against ν_s 's [39]. The present data appears to favor at the two σ level [40] the oscillation to ν_τ 's but cannot be taken as a conclusive proof.

Case B1.3:

Another possibility is to have the three active neutrinos bunched together with very minute mass differences such that $\nu_e - \nu_\mu$ oscillation explains the solar neutrino data and the $\nu_\mu - \nu_\tau$ oscillation explains the atmospheric puzzle. The LSND data can then be explained by including a sterile neutrino (or three of them) which is separated in mass from the three known neutrinos by an amount $\sqrt{\Delta m_{LSND}^2}$ and using indirect oscillation between the $\nu_e - \nu_\mu$ via the sterile neutrino state [41]. It has however been recently pointed out that this possibility may be marginally inconsistent with the observed up-down asymmetry in atmospheric neutrino data when combined with the CDHS and Bugey limits [42].

Let us study possible mass textures for four neutrinos. The general mass matrix in this case is a 4×4 symmetric matrix which has 10 nonzero entries if CP conservation is assumed. An interesting question to explore is: what the minimum number of parameters that are

necessary to fit observations. This would then isolate useful mass matrices with specific textures which may then lead to clues to their theoretical origin. For this purpose let us do a bit of parameter counting. If the solar neutrino puzzle is assumed to be solved by the small angle MSW mechanism, then there must be at least five parameters in the 4×4 symmetric neutrino mass matrix: three corresponding to $\Delta m_{S,A,L}^2$ and two small mixings θ_L and θ_S (where S, A, L denote respectively the solar, atmospheric and LSND experiments). If on the other hand the solar neutrino puzzle is solved via the vacuum oscillation mechanism, then one parameter is eliminated and one can make do with four parameters. Finally, if one adopts either the large angle MSW (between ν_e and ν_s) for solar neutrinos or the $\nu_\mu - \nu_s$ alternative is used to solve the atmospheric neutrino puzzle, then we have the additional relation between parameters i.e.

$$\frac{\Delta m_S^2}{\Delta m_A^2} \approx \frac{\Delta m_L^2}{\Delta m_L^2} \quad (22)$$

This reduces the number of parameters to three. An example of this type has recently been provided in Ref. [43].

Let us start with the simplest case with three parameters [43]. (Here we have used the basis $(\nu_s, \nu_e, \nu_\mu, \nu_\tau)$. It could be easily reshuffled to consider the $\nu_\mu - \nu_s$ oscillation possibility for atmospheric neutrinos.)

$$M = \begin{pmatrix} m_1 & \mu & m_1 - \epsilon & 0 \\ \mu & m_1 & 0 & m_1 - \epsilon \\ m_1 - \epsilon & 0 & m_1 & \mu \\ 0 & m_1 - \epsilon & \mu & m_1 \end{pmatrix}. \quad (23)$$

Here ϵ is the solar mass splitting and m_1 is the atmospheric mass splitting.

The next simplest case seems to involve only four parameters and has the following form [44]:

$$M = \begin{pmatrix} 0 & \mu_3 & 0 & 0 \\ \mu_3 & 0 & \epsilon & 0 \\ 0 & \epsilon & 0 & m \\ 0 & 0 & m & \delta \end{pmatrix}. \quad (24)$$

where we choose $\delta \ll m \simeq 1$ eV. However, since in this case the sterile neutrino has a smaller mass than the ν_e (as can be seen by diagonalizing the above mass matrix), the solution to the solar neutrino puzzle must involve the vacuum oscillation of the ν_e to ν_s . This can therefore be clearly tested once sufficient solar neutrino data accumulates (say for example from Super-Kamiokande and Borexino) in favor of or against the seasonal variation.

Going to one more parameter, we have the mass matrix which has one of the two following forms [45]:

$$M = \begin{pmatrix} \mu_1 & \mu_3 & 0 & 0 \\ \mu_3 & 0 & 0 & \epsilon \\ 0 & 0 & \delta & m \\ 0 & \epsilon & m & \pm\delta \end{pmatrix}. \quad (25)$$

In this case by an appropriate choice of μ_1 , the sterile neutrino can be made heavier than the electron neutrino. As a result, one can have useful MSW transition between the ν_e and ν_s that can help to solve the solar neutrino puzzle via the small angle MSW solution. This possibility has its characteristic predictions and it is expected that it can be tested in future colliders such as the muon collider. For discussion of CP violation in these models see Ref. [46].

B2. Six neutrino models

One obvious question that arises as we consider sterile neutrino models is “how many such neutrinos are there?”. Symmetry would suggest that there are three sterile neutrinos rather than one. In fact there are models of this type in the literature [48,47,49]. In one class of models, [48], the additional neutrinos (i.e. the fifth and the sixth) do not play any role in describing neutrino observations; but in the other two [47,49], they play an essential role. The profile of oscillation explanations in this case is very different and more symmetric in the sense that the both solar and atmospheric neutrino data are explained by maximal mixing angle vacuum oscillation between active and sterile neutrinos (i.e. solar via $\nu_e - \nu_{es}$ and atmospheric via $\nu_\mu - \nu_{\mu s}$ vacuum oscillations) [50]; the LSND results in this case is explained via generational mixing which is small like all other charged fermion mixings. The physics of this scheme is different from the previous ones as are the experimental tests. We will discuss typical theories for such scenarios in subsequent sections.

Before closing this section, it is worth pointing out that the introduction of the sterile neutrino is such a far reaching idea that attempts have been made to reconcile the LSND data without invoking oscillations in which case solar and atmospheric neutrino puzzles as well as the LSND data can be accommodated within the three standard neutrino framework. The idea is to look for consistent models where a sufficiently significant amplitude for the anomalous decay mode of muon $\mu^+ \rightarrow e^+ + \bar{\nu}_e + \bar{\nu}_\mu$ exists without conflicting with other known data to explain the 0.3% rate for the observed $\bar{\nu}_e$'s. However it has been pointed out [51] that the muonium to anti-muonium ($M - \bar{M}$) transition is related to the anomalous muon decay in several models (such as left-right models or supersymmetric models with R-parity breaking) in such a way that the existing experimental bounds from PSI [52] on $M - \bar{M}$ transition suppresses the anomalous muon decay amplitude far below that required to explain the LSND data. Thus barring some really drastic new idea to explain the LSND data as an anomalous muon decay, the confirmation of the LSND data in future would require the existence of additional one or more sterile neutrinos¹. It will then be dependent on the data as to which of the scenarios with extra sterile neutrinos will win in the end.

¹There have also been attempts to explain the atmospheric neutrino data by non-oscillation mechanisms [53] such as flavor changing interactions or neutrino decays etc. In a recent paper, Bergman et al [54] have argued that the first alternative runs into problem with large flavor changing effects in τ decays, which have not been observed and therefore that mechanism is theoretically not very plausible.

V. WHY NEUTRINO MASS NECESSARILY MEANS PHYSICS BEYOND THE STANDARD MODEL ?

As is wellknown, the standard model is based on the gauge group $SU(3)_c \times SU(2)_L \times U(1)_Y$ group under which the quarks and leptons transform as described in the Table I.

The electroweak symmetry $SU(2)_L \times U(1)_Y$ is broken by the vacuum expectation of the Higgs doublet $\langle H^0 \rangle = v_{wk} \simeq 246$ GeV, which gives mass to the gauge bosons and the fermions, all fermions except the neutrino. Thus the neutrino is massless in the standard model, at the tree level. There are several questions that arise at this stage. What happens when one goes beyond the above simple tree level approximation ? Secondly, do nonperturbative effects change this tree level result ? Finally, how will this result be modified when the quantum gravity effects are included ?

Table I

Field	gauge transformation
Quarks Q_L	$(3, 2, \frac{1}{3})$
Righthanded up quarks u_R	$(3, 1, \frac{2}{3})$
Righthanded down quarks d_R	$(3, 1, -\frac{2}{3})$
Lefthanded Leptons L	$(1, 2, -1)$
Righthanded leptons e_R	$(1, 1, -2)$
Higgs Boson \mathbf{H}	$(1, 2, +1)$
Color Gauge Fields G_a	$(8, 1, 0)$
Weak Gauge Fields W^\pm, Z, γ	$(1, 3 + 1, 0)$

Table caption: The assignment of particles to the standard model gauge group $SU(3)_c \times SU(2)_L \times U(1)_Y$.

The first and second questions are easily answered by using the B-L symmetry of the standard model. The point is that since the standard model has no $SU(2)_L$ singlet neutrino-like field, the only possible mass terms that are allowed by Lorentz invariance are of the form $\nu_{iL}^T C^{-1} \nu_{jL}$, where i, j stand for the generation index and C is the Lorentz charge conjugation matrix. Since the ν_{iL} is part of the $SU(2)_L$ doublet field and has lepton number +1, the above neutrino mass term transforms as an $SU(2)_L$ triplet and furthermore, it violates total lepton number (defined as $L \equiv L_e + L_\mu + L_\tau$) by two units. However, a quick look at the standard model Lagrangian convinces one that the model has exact lepton number symmetry both before and after symmetry breaking; therefore such terms can never arise in perturbation theory. Thus to all orders in perturbation theory, the neutrinos are massless. As far as the nonperturbative effects go, the only known source is the weak instanton effects. Such effects could effect the result if they broke the lepton number symmetry. One way to see if such breaking occurs is to look for anomalies in lepton number current conservation from triangle diagrams. Indeed $\partial_\mu j_\ell^\mu = cW\tilde{W} + c'B\tilde{B}$ due to the contribution of the leptons to the triangle involving the lepton number current and W 's or B 's. Luckily, it turns out that the anomaly contribution to the baryon number current nonconservation has also an identical form, so that the $B - L$ current j_{B-L}^μ is conserved to all orders in the gauge couplings. As a consequence, nonperturbative effects from the gauge sector cannot induce

$B - L$ violation. Since the neutrino mass operator described above violates also $B - L$, this proves that neutrino masses remain zero even in the presence of nonperturbative effects.

Let us now turn to the effect of gravity. Clearly as long as we treat gravity in perturbation theory, the above symmetry arguments hold since all gravity coupling respect $B - L$ symmetry. However, once nonperturbative gravitational effects e.g black holes and worm holes are included [55], there is no guarantee that global symmetries will be respected in the low energy theory. The intuitive way to appreciate the argument is to note that throwing baryons into a black hole does not lead to any detectable consequence except thru a net change in the baryon number of the universe. Since one can throw in an arbitrary number of baryons into the black hole, an arbitrary information loss about the net number of missing baryons would prevent us from defining a baryon number of the visible universe- thus baryon number in the presence of a black hole can not be an exact symmetry. Similar arguments can be made for any global charge such as lepton number in the standard model. A field theoretic parameterization of this statement is that the effective low energy Lagrangian for the standard model in the presence of black holes and worm holes etc must contain baryon and lepton number violating terms. In the context of the standard model, the only such terms that one can construct are nonrenormalizable terms of the form $LHLH/M_{Pl}$. After gauge symmetry breaking, they lead to neutrino masses. It however becomes immediately clear that these masses are not enough to understand present observations since they are at most of order $v_{wk}^2/M_{Pl} \simeq 10^{-5}$ eV [56] and as we discussed in the previous section, in order to solve the atmospheric neutrino problem, one needs masses at least three orders of magnitude higher.

Thus one must seek physics beyond the standard model to explain observed evidences for neutrino masses.

VI. SCENARIOS FOR SMALL NEUTRINO MASS WITHOUT RIGHT HANDED NEUTRINOS

There exist a number of extensions of the standard model that lead to neutrino masses. Even restricting ourselves to the cases where the neutrinos are Majorana particles there come to mind at least three different mechanisms for neutrino masses. All these models have lepton number violation built into them and therefore lead to a plethora of new phenomenological tests. The existence or nonexistence of the right handed neutrinos divides these models into two broad classes: one class that uses right handed neutrinos and another that does not. In this section we consider models that do not introduce right handed neutrinos to understand small neutrino masses.

As discussed in section II, the electrical neutrality of the neutrino allows for the existence of two types of mass terms consistent with Lorentz invariance: the Majorana mass, which violates lepton number but does not require the inclusion of a right handed neutrino and the Dirac (or combined Dirac-Majorana) mass term which requires the existence of a right handed neutrino. In this subsection we consider models where the neutrino Majorana mass arises without the right handed neutrino. The argument of section IV then implies that there must be violation of the B-L quantum number either by interactions or by the vacuum state. This leads to two classes of models which we discuss below.

A. Radiative generation of neutrino masses

There are two classes of models where introduction of lepton number violating interactions leads to radiative generation of small neutrino masses. One is the Zee model [57] and the second one is the Babu model [58]. Let us first discuss the Zee model. In this case, one adds a charged $SU(2)_L$ -singlet field η^+ to the standard model along with a second Higgs doublet H' . This allows the following additional Yukawa couplings beyond those present in the standard model:

$$\mathcal{L}_Y(\eta) = \Sigma_{\alpha\beta} f_{\alpha\beta} L_\alpha^T C^{-1} L_\beta \eta^+ + \text{h.c.} \quad (26)$$

Note that the coupling f is antisymmetric in the family indices α and β . Due to the presence of the extra Higgs field, there are also additional terms in the Higgs potential but the one of interest in connection with the neutrino masses is $\eta^* H^T \tau_2 H'$. It is then easy to see that while the neutrino masses vanish at the tree level they arise from one loop diagrams due to the exchange of η fields in combination with the second Higgs (see Fig. 1). The typical strength of these diagrams is of order $\frac{m_\ell^2 f}{16\pi^2 M}$ [59]. The dominant contribution to the $\nu_{e,\mu}$ mass in these models comes from the τ lepton in the loop and can be estimated to be of order 1-10 keV for $f \simeq 10^{-2}$ and the $\eta H H'$ coupling also of order 10^{-2} - 10^{-1} . These values for the masses are much bigger than being contemplated in the context of neutrino puzzles. Thus in the opinion of this author, Zee model cannot account for the present observations without further ado.

Let us now pass to the second class of models [58]. In this case one adds to the standard model the fields η^+ as before and a doubly charged field h^{++} but not a second Higgs doublet. The new Yukawa couplings of the model then are given by (repeated indices are summed):

$$\mathcal{L}_Y(\eta, h) = f_{\alpha\beta} \eta L_\alpha L_\beta + f'_{\alpha\beta} h^{++} e_{\alpha,R}^T C^{-1} e_{\beta,R} + \text{h.c.} \quad (27)$$

The Higgs potential has the following term that is of interest in connection with the neutrino masses i.e. $\lambda'' v_{wk} h^* \eta \eta$. The model leads to nonzero contribution to neutrino masses at the two loop level via the diagrams of figure 2.

The typical estimate for the neutrino mass in this case is $\sim \frac{f^2 f' m_\ell^2 \lambda''}{(16\pi^2)^2 M}$. For $f \sim 10^{-1.5} \sim f' \sim \lambda''$, the two loop contribution to neutrino mass is of order 0.01 eV which is of the right order of magnitude for solving the present neutrino puzzles. No excessive fine tuning of couplings is needed. In building realistic models, one has to however pay attention to possible flavor changing neutral current effects such as the $\mu \rightarrow e \gamma$ decay for which there exist rather stringent constraints from the recent MEGA experiment [60]: $B(\mu \rightarrow e + \gamma) \leq 10^{-11}$. This will put constraints on the parameters as follows: $f_{13} f_{23} \leq 10^{-5}$ and $(f'_{13} f'_{23} + f'_{12} f'_{11,22}) \leq 10^{-5}$. We do not get into detailed model building for this case.

One process that distinguishes the second class of models from the first is the muonium to antimuonium ($M - \bar{M}$) oscillation which is mediated at the tree level by the doubly charged bosons [61]. The present limit on the strength of the effective four-Fermi interaction describing $M - \bar{M}$ transition is given by $G_{M-\bar{M}} \leq 3 \times 10^{-3} G_F$ [52]. For a 100 GeV h^{++} boson, this implies that $h_{ee} h_{\mu\mu} \leq 3 \times 10^{-4}$, which is a nontrivial constraint on the model. Since for the neutrino masses to be in the interesting range, the Higgs masses should not be more than a TeV, further improvement of the $M - \bar{M}$ transition limit can provide important information on this model.

B. High mass Higgs triplet and induced neutrino masses

Another way to generate nonzero neutrino masses without using the righthanded neutrino is to include in the standard model an $SU(2)_L$ triplet Higgs field with $Y = 2$ so that the electric charge profile of the members of the multiplet is given as follows: $(\Delta^{++}, \Delta^+, \Delta^0)$. This allows an additional Yukawa coupling of the form $f_L L^T \tau_2 \tau L \cdot \Delta$, where the Δ^0 couples to the neutrinos. Clearly Δ field has $L = 2$. When Δ^0 field has a nonzero vev, it breaks lepton number by two units and leads to Majorana mass for the neutrinos. There are two questions that arise now: one, how does the vev arise in a model and how does one understand the smallness of the neutrino masses in this scheme. There are two answers to the first question: One can maintain exact lepton number symmetry in the model and generate the vev of the triplet field via the usual “mexican hat” potential. There are two problems with this case. This leads to the massless triplet Majoron [62] which has been ruled out by LEP data on Z-width. Though it is now redundant it may be worth pointing out that in this model smallness of the neutrino mass is not naturally understood.

There is however another way to generate the induced vev keeping a large but positive mass (M_Δ) for the triplet Higgs boson and allowing for a lepton number violating coupling $M\Delta^* H H$. In this case, minimization of the potential induces a vev for the Δ^0 field when the doublet field acquires a vev:

$$v_T \equiv \langle \Delta^0 \rangle = \frac{M v_{wk}^2}{M_\Delta^2} \quad (28)$$

Since the mass of the Δ field is invariant under $SU(2)_L \times U(1)_Y$, it can be very large connected perhaps with some new scale of physics. If we assume that $M_\Delta \sim M \sim 10^{13}$ GeV or so, we get $v_T \sim$ eV. Now in the Yukawa coupling $f_L L^T \tau_2 \tau L \cdot \Delta$, since the Δ^0 couples to the neutrinos, its vev leads to a neutrino mass in the eV range or less depending on the value of the Yukawa couplings [63]. We will see later when we discuss the seesaw models that unlike those models, the neutrino mass in this case is not hierarchically dependent on the charged fermion masses. Another point worth emphasizing is that unlike the previous radiative scenarios, this model is more in the spirit of grand unification and can in fact be implemented in models [1] such as those based on the $SU(5)$ group where there is no natural place for the right handed neutrino.

C. Baryogenesis problem in models without right handed neutrinos

While strictly the models just discussed provide a way to understand the small neutrino masses without fine tuning, they may face problems in explaining the origin of matter in the universe. Let us first consider the triplet vev models. It was shown by Ma and Sarkar [63], that the decay of triplet Higgs to leptons provides a way to generate enough baryons in the model. However for that to happen, one must satisfy one of Sakharov’s three condition that the decay particle which leads to baryon or lepton asymmetry must be out of equilibrium. This requires that when the temperature of the universe equals the decaying particle’s mass, its decay rate must be smaller than the expansion rate of the universe i.e.

$$\frac{f_L^2 M_\Delta}{12\pi} < \sqrt{g^*} \frac{M_\Delta^2}{M_{Pl}} \quad (29)$$

This implies a lower limit on the mass $M_\Delta \geq \frac{f_L^2 M_{P\ell}}{12\pi\sqrt{g^*}}$. For $f_L \sim 10^{-1}$ as would be required by the atmospheric neutrino data, one gets conservatively, $M_\Delta \geq 10^{13}$ GeV. The problem with such a large mass arises from the fact that in an inflationary model of the universe, the typical reheating temperature dictated by the gravitino problem of supergravity is at most 10^9 GeV. Thus there is an inherent conflict [64] between the standard inflationary picture of the universe and the baryogenesis in the simple triplet model for neutrino masses.

As far as the radiative models go, they have explicit lepton violating terms in the Lagrangian which are significant enough to keep these interactions in equilibrium for all interesting low temperatures ($T < 10^{12}$ GeV or so). So it is not clear how one would ever generate any baryon number in this class of models.

VII. SEESAW MECHANISM AND LEFT RIGHT SYMMETRIC UNIFICATION MODELS FOR SMALL NEUTRINO MASSES

A very natural and elegant way to generate neutrino masses is to include the right handed neutrinos in the standard model. However the inclusion of the right handed neutrinos transforms the dynamics of gauge models in a profound way. To clarify what we mean, note that in the standard model (that does not contain a ν_R) the $B-L$ symmetry is only linearly anomaly free i.e. $Tr[(B-L)Q_a^2] = 0$ where Q_a are the gauge generators of the standard model but $Tr(B-L)^3 \neq 0$. This means that $B-L$ is only a global symmetry and cannot be gauged. However as soon as the ν_R is added to the standard model, one gets $Tr[(B-L)^3] = 0$ implying that the B-L symmetry is now gaugeable and one could choose the gauge group of nature to be either $SU(2)_L \times U(1)_{I_{3R}} \times U(1)_{B-L}$ or $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$, the latter being the gauge group of the left-right symmetric models [65]. Furthermore the presence of the ν_R makes the model quark lepton symmetric and leads to a Gell-Mann-Nishijima like formula for the electric charges [66] i.e.

$$Q = I_{3L} + I_{3R} + \frac{B-L}{2} \quad (30)$$

The advantage of this formula over the charge formula in the standard model is that in this case all entries have a physical meaning. It also provides a natural understanding of Majorana nature of neutrinos as can be seen by looking at the distance scale where the $SU(2)_L \times U(1)_Y$ symmetry is valid but the left-right gauge group is broken. In that case, one gets

$$\begin{aligned} \Delta Q &= 0 = \Delta I_{3L} : \\ \Delta I_{3R} &= -\Delta \frac{B-L}{2} \end{aligned} \quad (31)$$

We see that if the Higgs fields that break the left-right gauge group carry righthanded isospin of one, one must have $|\Delta L| = 2$ which means that the neutrino mass must be Majorana type and the theory will break lepton number by two units.

Let us now proceed to discuss the left-right symmetric model and demonstrate how the seesaw mechanism emerges in this model.

The gauge group of the theory is $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ with quarks and leptons transforming as doublets under $SU(2)_{L,R}$. In Table 2, we present transformation properties of the quark, lepton and Higgs fields of the model under the gauge group.

Table II

Fields	$SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ representation
Q_L	$(2, 1, +\frac{1}{3})$
Q_R	$(1, 2, \frac{1}{3})$
L_L	$(2, 1, -1)$
L_R	$(1, 2, -1)$
ϕ	$(2, 2, 0)$
Δ_L	$(3, 1, +2)$
Δ_R	$(1, 3, +2)$

Table caption Assignment of the fermion and Higgs fields to the representation of the left-right symmetry group.

The first task is to specify how the left-right symmetry group breaks to the standard model i.e. how one breaks the $SU(2)_R \times U(1)_{B-L}$ symmetry so that the successes of the standard model including the observed predominant V-A structure of weak interactions at low energies is reproduced. Another question of naturalness that also arises simultaneously is that since the charged fermions and the neutrinos are treated completely symmetrically (quark-lepton symmetry) in this model, how does one understand the smallness of the neutrino masses compared to that of the other fermions.

It turns out that both the above problems of the LR model have a common solution. The process of spontaneous breaking of the $SU(2)_R$ symmetry that suppresses the V+A currents at low energies also solves the problem of ultralight neutrino masses. To see this let us write the Higgs fields in terms of its components:

$$\Delta = \begin{pmatrix} \Delta^+/\sqrt{2} & \Delta^{++} \\ \Delta^0 & -\Delta^+/\sqrt{2} \end{pmatrix}; \quad \phi = \begin{pmatrix} \phi_1^0 & \phi_2^+ \\ \phi_1^- & \phi_2^0 \end{pmatrix} \quad (32)$$

All these Higgs fields have Yukawa couplings to the fermions given symbolically as below.

$$\begin{aligned} \mathcal{L}_Y = & h_1 \bar{L}_L \phi L_R + h_2 \bar{L}_L \tilde{\phi} L_R \\ & + h'_1 \bar{Q}_L \phi Q_R + h'_2 \bar{Q}_L \tilde{\phi} Q_R \\ & + f(L_L L_L \Delta_L + L_R L_R \Delta_R) + h.c. \end{aligned} \quad (33)$$

The $SU(2)_R \times U(1)_{B-L}$ is broken down to the standard model hypercharge $U(1)_Y$ by choosing $\langle \Delta_R^0 \rangle = v_R \neq 0$ since this carries both $SU(2)_R$ and $U(1)_{B-L}$ quantum numbers. It gives mass to the charged and neutral righthanded gauge bosons i.e. $M_{W_R} = gv_R$ and $M_{Z'} = \sqrt{2}gv_R \cos\theta_W / \sqrt{\cos 2\theta_W}$. Thus by adjusting the value of v_R one can suppress the right handed current effects in both neutral and charged current interactions arbitrarily leading to an effective near maximal left-handed form for the charged current weak interactions at low energies.

The fact that at the same time the neutrino masses also become small can be seen by looking at the form of the Yukawa couplings. Note that the f-term leads to a mass for the right handed neutrinos only at the scale v_R . Next as we break the standard model symmetry by turning on the vev's for the ϕ fields as $Diag <\phi> = (\kappa, \kappa')$, we not only give masses to the W_L and the Z bosons but also to the quarks and the leptons. In the neutrino sector the above Yukawa couplings after $SU(2)_L$ breaking by $<\phi> \neq 0$ lead to the Dirac masses for the neutrino connecting the left and right handed neutrinos. In the two component neutrino language, this leads to the following mass matrix for the ν, N (where we have denoted the left handed neutrino by ν and the right handed component by N).

$$M = \begin{pmatrix} 0 & h\kappa \\ h\kappa & fv_R \end{pmatrix} \quad (34)$$

By diagonalizing this 2×2 matrix, we get the light neutrino eigenvalue to be $m_\nu \simeq \frac{(h\kappa)^2}{fv_R}$ and the heavy one to be fv_R . Note that typical charged fermion masses are given by $h\kappa$ etc. So since $v_R \gg \kappa, \kappa'$, the light neutrino mass is automatically suppressed. This way of suppressing the neutrino masses is called the seesaw mechanism [2]. Thus in one stroke, one explains the smallness of the neutrino mass as well as the suppression of the V+A currents².

In deriving the above seesaw formula for neutrino masses, it has been assumed that the vev of the lefthanded triplet is zero so that the $\nu_L \nu_L$ entry of the neutrino mass matrix is zero. However, in most explicit models such as the left-right model which provide an explicit derivation of this formula, there is an induced vev for the Δ_L^0 of order $<\Delta_L^0> = v_T \simeq \frac{v_{wk}^2}{v_R}$. In the presence of this term the seesaw formula undergoes a fundamental change. Let us therefore distinguish between two types of seesaw formulae:

Type I seesaw formula

$$M_\nu \simeq -M_D^T M_{N_R}^{-1} M_D \quad (35)$$

where M_D is the Dirac neutrino mass matrix and $M_{N_R} \equiv fv_R$ is the right handed neutrino mass matrix in terms of the Δ Yukawa coupling matrix f .

Type II seesaw formula

$$M_\nu \simeq f \frac{v_{wk}^2}{v_R} - M_D^T M_{N_R}^{-1} M_D \quad (36)$$

Note that in the type I seesaw formula, what appears is the square of the Dirac neutrino mass matrix which in general expected to have the same hierarchical structure as the

² There is an alternative class of left-right symmetric models where small neutrino masses can arise from radiative corrections, if one chooses only doublet Higgses to break the gauge symmetry and heavy vector-like charged fermions to understand fermion mass hierarchies. In these models the neutrinos are Dirac particles [67].

corresponding charged fermion mass matrix. In fact in some specific GUT models such as SO(10), $M_D = M_u$. This is the origin of the common statement that neutrino masses given by the seesaw formula are hierarchical i.e. $m_{\nu_e} \ll m_{\nu_\mu} \ll m_{\nu_\tau}$ and even a more model dependent statement that $m_{\nu_e} : m_{\nu_\mu} : m_{\nu_\tau} = m_u^2 : m_c^2 : m_t^2$.

On the other hand if one uses the type II seesaw formula, there is no reason to expect a hierarchy and in fact if the neutrino masses turn out to be degenerate as discussed before as one possibility, one way to understand this may be to use the type II seesaw formula.

Type III seesaw formula

While the above seesaw formulae are extremely helpful in building models with ultralight neutrino masses, there are circumstances where they are not helpful. An obvious example is one where the righthanded neutrino mass matrix is singular due to additional symmetries (e.g. $L_e - L_\mu - L_\tau$ say), in which case its inverse does not exist and alternative ways to understand lightness of neutrinos must be explored. There is one such mechanism in literature which we will call type III seesaw which involves a 3×3 seesaw pattern [68] rather than the just described 2×2 one. For one generation, it involves three fermions (ν, N, S), where ν and N are the usual left and right handed neutrinos and S is a singlet neutrino (singlet under the left-right group). The relevant seesaw matrix is given by

$$\mathcal{M}_\nu = \begin{pmatrix} 0 & m_D & 0 \\ m_D & 0 & M \\ 0 & M & \mu \end{pmatrix} \quad (37)$$

We assume that $\mu, m_D \ll M$ in which case the three eigenvalues are given by

$$m_\nu \sim \frac{m_D^2 \mu}{M^2} \quad (38)$$

Its generalization to the case when each element is a matrix is obvious. The important point here is to note that the matrix μ is in the numerator. As a result, if we want the light neutrino mass matrix to be singular (say $L_e - L_\mu - L_\tau$ invariant), then this can be built into the matrix μ and one can then use the seesaw formula given here to understand the smallness of the neutrino masses. In the context of left-right models this kind of a structure for neutrino masses arises if the triplets above are replaced by $B - L = 1$ doublets and we supplement the model with one singlet fermion per generation. (*Show this by explicit construction and work out the example with $L_e - L_\mu - L_\tau$ symmetry*). Another feature of the type III seesaw is that one could choose the B-L breaking scale to be much lower than in the case of the types I and II.

Let us now address the question: to what extent one can understand the details of the neutrino masses and mixings using the seesaw formulae. The answer to this question is quite model dependent. While one can say that there exist many models which fit the observations, none (except a few) are completely predictive. The problem in general is that the seesaw formula of type I has 12 parameters which is why its predictive power is so limited. One number that is predicted in a class of seesaw models based on the SO(10) group that embodies the left-right symmetric unification model or the SU(4)-color is the tau neutrino mass. In this class of models, one maintains the quark lepton symmetry in the

leading order so that one has the relation $m_t(M_U) = m_{\nu_\tau}^D$. The tau neutrino mass is then given by the seesaw formula to be $m_{\nu_\tau} \simeq \frac{m_t^2}{M_{N_{3R}}}$. In one class of string inspired models, where the $SU(2)_R$ symmetry and the GUT symmetry break at the same scale, the right handed neutrino masses are generated by nonrenormalizable operators and they are given in the simplest approximation by $\frac{fv_R^2}{M_{Pl}} \simeq f 10^{14}$ GeV. If $f = 1$, then one gets $m_{\nu_\tau} \simeq 0.03$ eV which is the right value to fit the atmospheric neutrino data. The rest of the data such as the maximal mixing with the muon neutrino etc are not predicted without further assumptions. Even the prediction of the ν_τ mass requires an assumption that the Yukawa coupling f must be unity.

A. SO(10) realization of the seesaw mechanism

The most natural grand unified theory for the seesaw mechanism is the SO(10) model. In this paragraph, some of the salient features of this realization are summarized. From the previous paragraph, we learn that the simplest left right model with B-L=2 triplets provides a direct realization of the seesaw mechanism. In the context of the SO(10) model, the first point to note is that the **16**-dimensional spinor representation contains all the fermions of each generation in the standard plus the right handed neutrino. Thus the right handed neutrino is automatic in the SO(10) model. Secondly, in order to break the B-L symmetry present in the SO(10) group, one may use either the Higgs multiplets in **16** or **126** dimensional representation. Under the left-right symmetric group $SU(2)_L \times SU(2)_R \times U(1)_{B-L} \times SU(3)_c$, these fields decompose as follows:

$$\begin{aligned} \mathbf{16}_H &= (2, 1, 1/3, 3) \oplus (1, 2, -1/3, 3^*) \oplus (2, 1, -1, 1) \oplus (1, 2, +1, 1) \\ \mathbf{126} &= (1, 1, -2/3, 3) \oplus (1, 1, 2/3, 3^*) \oplus (3, 1, -2, 1) \oplus (1, 3, +2, 1) + \dots \end{aligned} \quad (39)$$

where the ellipses denote other multiplets that have no role in neutrino mass discussion. In order to break the B-L symmetry it is the last entry in each of the multiplets whose neutral elements need to pick up a large vev. Note however that the first one (i.e. the $\mathbf{16}_H$) does not have any renormalizable coupling with the **16** spinors which contain the ν_R whereas there is a renormalizable SO(10) invariant coupling of the form $\mathbf{16}\mathbf{16}\mathbf{126}$. for the second multiplet. Therefore if we decided to stay with the renormalizable model, then one would need a **126** dimensional representation to implement the seesaw mechanism whereas if we used the **16** to break the B-L symmetry, we would require nonrenormalizable couplings of the form $\mathbf{16}^2\mathbf{16}_H^2/M_{Pl}$. This has important implications for the B-L scale. In the former case, the B-L breaking scale is at an intermediate level such as $\sim 10^{13}$ GeV or so whereas in the latter case, we can have B-L scale coincide with the GUT scale of 2×10^{16} GeV as in the typical SUSYGUT models [71].

Another advantage of SO(10) models in understanding neutrino masses is that if one uses only the **10** dimensional representation for giving masses to the quarks and leptons, one has the up quark mass matrix M_u being equal to the Dirac mass matrix of the neutrinos which goes into the seesaw formula. As a result, if we work in a basis where the up quark masses are diagonal so that all CKM mixings come from the down mass matrix, then the number of arbitrary parameters in the seesaw formula goes down from 12 to 6. Thus even though one cannot predict neutrino masses and mixings, the parameters of the theory get

fixed by their values as inputs. This may then be testable thru its other predictions. In this model however, there are tree level mass relations in the down sector such as $\frac{m_d}{m_s} = \frac{m_e}{m_\mu}$ which are renormalization group invariant and are in disagreement with observations. It may be possible in supersymmetric models to generate enough one loop corrections out of the supersymmetry breaking terms (nonuniversal) to save the situation.

There is one very special cases where all 12 of these parameters are predicted by the quark and lepton masses [69]. The Higgs content of this model is only one **10** and one **126**. Their Yukawa couplings involve only nine parameters all of which along with input vevs of the doublets in both these multiplets are fixed by the quark masses, mixings and the three charged lepton masses. The important point is that the same **126** responsible for the fermion masses also has a vev along the $\nu_R \nu_R$ directions so that it generates the right handed neutrino mass matrix. Thus in the light neutrino sector it is a completely parameter free model. But this minimal SO(10) model cannot fit both the solar and the atmospheric neutrino data and is therefore ruled out. Recently other SO(10) models have been considered where under different assumptions, the atmospheric and solar neutrino data can be explained together [70]. These models have many interesting features, which we do not go into here for lack of space.

In the minimal supersymmetric left-right model, an analogous situation happens where the neutrino Dirac masses are found to be equal to the charged lepton masses [72]. Thus in this model too, one has only six parameters to describe the neutrino sector and once the neutrino data is fitted all parameters in the model are fixed so that one has predictions that can be tested. For instance, it has been emphasized in Ref. [72] that there is a prediction for the $B(\tau \rightarrow \mu + \gamma)$ in this model that is about two orders of magnitude below the present limits [73] and could therefore be used to test the model.

There is yet another class of models where by assigning $U(1)$ charges to the fermions of the standard model as well as the ν_R fields [74], one restricts the Dirac as well as Majorana mass matrices (for the ν_R). One then assumes that the $U(1)$ charges originate from strings or some high scale physics.

Finally let us comment that in models where the light neutrino mass is understood via the seesaw mechanism that uses heavy righthanded neutrinos, there is a very simple mechanism for the generation of baryon asymmetry of the universe. Since the righthanded neutrino has a high mass, it decays at a high temperature to generate a lepton asymmetry [75] and this lepton asymmetry is converted to baryon asymmetry via the sphaleron effects [76] at lower temperature. It also turns out that one of the necessary conditions for sufficient leptogenesis is that the right handed neutrinos must be heavy as is required by the seesaw mechanism. To see this note that one of Sakharov conditions for leptogenesis is that the right handed neutrino decay must be slower than the expansion rate of the universe at the temperature $T \sim M_{N_R}$. The corresponding condition is:

$$\frac{h_\ell^2 M_{N_R}}{16\pi} \leq \sqrt{g^*} \frac{M_{N_R}^2}{M_{P\ell}} \quad (40)$$

This implies that $M_{N_R} \geq \frac{h_\ell^2 M_{P\ell}}{16\pi\sqrt{g^*}}$. For the second generation, it implies that $M_{N_{2R}} \geq 10^{13}$ GeV and for the third generation a value even higher. Note that these are above the inflation reheating upper bound alluded to before. However for the first generation, it is about 10^8 GeV so that there is no conflict with the gravitino bound on the reheating temperature.

Incidentally, the leptogenesis condition also imposes limits on the matrix elements of the right handed neutrino mass, thereby reducing the arbitrariness of the seesaw predictions slightly.

VIII. NATURALNESS OF DEGENERATE NEUTRINOS

In this section we like to discuss some issues related to the degenerate neutrino hypothesis. Recall that this is the only scenario that fits all observations if one does not include LSND (say it is not confirmed by MiniBooNE) and the universe has 10% to 20% of its matter in the form of neutrinos. Thus it is appropriate to discuss how such models can arise in theoretical schemes and how stable they are under radiative corrections.

The first point already alluded to before and first made in [20] is that degenerate neutrinos arise naturally in models that employ the type II seesaw since the first term in the mass formula is not connected to the charged fermion masses. One way that has been discussed is to consider schemes where one uses symmetries such as SO(3) or SU(2) or permutation symmetry S_4 [77] so that the Majorana Yukawa couplings f_i are all equal. This then leads to the dominant contribution to all neutrinos being equal. This symmetry however must be broken in the charged fermion sector in order to explain the observed quark and lepton masses. Such models consistent with known data have been constructed based on SO(10) as well as other groups. The interesting point about the SO(10) realization is that the dominant contributions to the Δm^2 's in this model comes from the second term in the type II seesaw formula which in simple models is hierarchical. It is of course known that if the MSW solution to the solar neutrino puzzle is the right solution (or an energy independent solution), then we have $\Delta m_{solar}^2 \ll \Delta m_{ATMOS}^2$. In fact if we use the fact true in SO(10) models that $M_u = M_D$, then we have $\Delta m_{ATMOS}^2 \simeq m_0 \frac{m_t^2}{fv_R}$ and $\Delta m_{SOLAR}^2 \simeq m_0 \frac{m_c^2}{fv_R}$ where m_0 is the common mass for the three neutrinos. It is interesting that for $m_0 \sim \text{few eV}$ and $fv_R \approx 10^{15} \text{ GeV}$, both the Δm^2 's are close to the required values.

An interesting theoretical issue about these models has been raised in several recent papers [78]. It has been noted that even though one may have tree level models with a degenerate neutrino spectrum, it is not clear that this mass degeneracy will survive the radiative corrections. In fact it has been convincingly argued that in models with maximal mixings the departure from degeneracy may be significant. This provides a further challenge to model building and one must ensure that should maximal mixing models win the day, the degeneracy is preserved upto at least to two loop.

IX. THEORETICAL UNDERSTANDING OF THE STERILE NEUTRINO

If the existence of the sterile neutrino becomes confirmed say, by a confirmation of the LSND observation of $\nu_\mu - \nu_e$ oscillation or directly by SNO neutral current data to come in the early part of the next century, a key theoretical challenge will be to construct an underlying theory that embeds the sterile neutrino along with the others with appropriate mixing pattern, while naturally explaining its ultralightness.

If a sterile neutrino was introduced into the standard model, the gauge symmetry would not forbid its bare mass, implying that there would be no reason for its mass to be small. It is

a common experience in physics that if a particle has mass lighter than normally expected on the basis of known symmetries, then it is an indication for the existence of new symmetries. This line of reasoning has been pursued in recent literature to understand the ultralightness of the sterile neutrino by using new symmetries beyond the standard model.

There are several suggestions for this new symmetry that might help us to accomodate an ultralight ν_s and in the process lead us to new classes of extensions of physics beyond the standard model. We will consider two examples (i) one based on the E_6 grand unification model and (ii) another based on the possible existence of mirror matter in the universe. There are also other theoretical models for the sterile neutrino that we involve different assumptions [79] and we do not discuss them here.

A completely different way to understand the ultralightness of the sterile neutrino is to introduce large extra dimensions in a Kaluza-Klein framework [80] and not rely on any symmetries. In these models, the sterile neutrino is supposed to live in the bulk; therefore if its Lagrangian mass is zero, then the mass of the first Kaluza-Klein mode is inversely proportional to the size of the extra dimension. Thus the extra dimension size of order of a millimeter would lead to sterile neutrino masses in the range of milli eV's which are of interest for solar neutrino oscillations. There have been recent attempts [81] to build realistic models using a slight variation of this idea [82]. This class of models are distinguished from others in their prediction of infinite tower of sterile neutrinos, which at the moment seems perfectly consistent with observations. Eventually, this feature may provide a way to test such theories. We do not elaborate on these models any further.

X. E_6 MODEL FOR THE STERILE NEUTRINO

E_6 [83] is an interesting and viable unification group beyond the $SO(10)$ group and as such has been extensively discussed in literature [84]. In this model, matter belongs to the **27** dimensional representation of the E_6 group which under its $SO(10) \times U(1)$ subgroup decomposes to $\mathbf{16}_{+1} \oplus \mathbf{10}_{-2} \oplus \mathbf{1}_{+4}$ (the subscripts represent the $U(1)$ charges). The **16** is well known to contain the left and the right handed neutrinos (to be denoted by us as ν_i and ν_i^c , i being the family index). The **10** contains two neutral colorless fermions which behave like neutrinos but are $SU(2)_L$ doublets and the last neutral colorless fermion in the **27**, which we identify as the sterile neutrino is the one contained in the $SO(10)$ singlet multiplet (denoted by ν_{is}). This has all the properties desired of a sterile neutrino. Thus in this model there are three sterile neutrinos which will be denoted by the corresponding flavor label and we will show first how the small mass for both the active and the sterile neutrino results from a generalization of the seesaw mechanism as a consequence of the symmetries of the group [85]. Furthermore, we will see how as a consequence of the smallness of the Yukawa couplings of the standard model, we will not only get maximal mixing between the active and the sterile neutrinos of each generation but also the necessary ultra-small Δm^2 needed in the vacuum oscillation solution to the solar neutrino puzzle without fine tuning of parameters [49]. Thus in this model, both the solar and the atmospheric neutrino puzzles are solved by the maximal vacuum oscillation of active to sterile neutrinos.

In general in this model, we will have for each generation a 5×5 “neutrino” mass matrix. To give an essence of the basic steps that lead to the seesaw mechanism, it is necessary to describe the symmetry breaking of E_6 . We work with a supersymmetric E_6 and use three

pairs of $\mathbf{27} + \overline{\mathbf{27}}$ representations and one $\mathbf{78}$ -dim. field for symmetry breaking. The pattern of symmetry breaking is as follows:

- 1) $\langle 27_1 \rangle$ and $\langle \overline{27}_1 \rangle$ have GUT scale vevs in the $\text{SO}(10)$ singlet direction.
- 2) $\langle 27_{16} \rangle$ and $\langle \overline{27}_{16} \rangle$ have GUT scale vevs in the ν and ν^c directions respectively. They break $\text{SO}(10)$ down to $\text{SU}(5)$.
- 3) The $\langle 78_{[1,45]} \rangle$ completes the breaking of $\text{SU}(5)$ to the standard model gauge group at the GUT scale. We assume the VEVs reside both in the adjoint and in the singlet of $\text{SO}(10)$.
- 4) $\langle 27_{10} \rangle$ and $\langle \overline{27}_{10} \rangle$ contain the Higgs doublets of the MSSM. It is assumed that H_u and H_d are both linear combinations arising partially from the $\langle 27_{10} \rangle$ and partially from the $\langle \overline{27}_{10} \rangle$.

In addition to the above there is another field labelled by $27'$ whose ν^c component mixes with a singlet S and one linear combination of this pair remains light below the GUT scale. As a consequence of radiative symmetry breaking this picks up a VEV at the electroweak scale. This was shown in Ref. [49]. The remaining components of $27'$ have GUT scale mass. Let us now write down the relevant terms in the superpotential that lead to a 5×5 “neutrino” mass matrix of the form we desire. To keep matters simple let us ignore generation mixings, which can be incorporated very trivially.

$$W = \lambda_i \psi_i \psi_i 27_{10} + f_i \psi_i \psi_i 27' + \frac{\alpha_i}{M_{P\ell}} \psi_i \psi_i 27_1 78_{[1,45]} + \frac{\gamma_i}{M_{P\ell}} \psi_i \psi_i \overline{27}_{16} \overline{27}_{16} \quad (41)$$

We have shown only a subset of allowed terms in the theory that play a role in the neutrino mass physics and believe that it is reasonable to assume a discrete symmetry (perhaps in the context of a string model) that would allow only this subset. In any case since we are dealing with a supersymmetric theory, radiative corrections will not generate any new terms in the superpotential.

Note that in Eq. (41), since it is the first term that leads to lepton and quark masses of various generations, it carries a generation label and obeys a hierarchical pattern, whereas the f_i 's not being connected to known fermion masses need not obey a hierarchical pattern. We will from now on assume that each $f_i \approx 1$.

After substituting the vevs for the Higgs fields in the above equation, we find a 5×5 mass matrix of the following form for the neutral lepton fields of each generation in the basis $(\nu, \nu_s, \nu^c, E_u^0, E_d^0)$:

$$M = \begin{pmatrix} 0 & 0 & \lambda_i v_u & f_i v' & 0 \\ 0 & 0 & 0 & \lambda_i v_d & \lambda_i v_u \\ \lambda_i v_u & 0 & M_{\nu^c, i} & 0 & 0 \\ f_i v' & \lambda_i v_d & 0 & 0 & M_{10, i} \\ 0 & \lambda_i v_u & 0 & M_{10, i} & 0 \end{pmatrix} \quad (42)$$

Here $M_{\nu^c, i}$ is the mass of the right handed neutrino and $M_{10, i}$ is the mass of the entire $\mathbf{10}$ -plet in the $\mathbf{27}$ matter multiplet. Since $\mathbf{10}$ contains two full $\text{SU}(5)$ multiplets, gauge coupling unification will not be effected even though we choose its mass to be below the GUT scale.

Note that the 3×3 mass matrix involving the (ν^c, E_u^0, E_d^0) have superheavy entries and will therefore decouple at low energies. Their effects on the spectrum of the light neutrinos will be dictated by the seesaw mechanism [2]. The light neutrino mass matrix involving ν_i, ν_{is} can be written down as:

$$M_{light} \simeq \frac{1}{M_{\nu^c,i}} \begin{pmatrix} \lambda_i v_u & f_i v' & 0 \\ 0 & \lambda_i v_d & \lambda_i v_u \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & \epsilon \\ 0 & \epsilon & 0 \end{pmatrix} \begin{pmatrix} \lambda_i v_u & 0 \\ f_i v' & \lambda_i v_d \\ 0 & \lambda_i v_u \end{pmatrix} \quad (43)$$

where $\epsilon_i = M_{10,i}/M_{\nu^c,i}$. Note that ϵ_i is expected to be of order one. This leads to the 2×2 mass matrix for the (ν, ν_c) fields of each generation the form,

$$M_i = m_{0i} \begin{pmatrix} \lambda_i^2 & \lambda_i \bar{f}_i \\ \lambda_i \bar{f}_i & \lambda_i^2 \bar{\epsilon}_i \end{pmatrix} \quad (44)$$

Here $m_{0i} = \frac{v_u^2}{M_{\nu^c,i}}$, $\bar{f}_i = f_i \epsilon_i v' / v_u$, and $\bar{\epsilon}_i = 2\epsilon_i \cot \beta$. Taking $M_{Pl} \sim 10^{19} GeV$, $M_{GUT} \sim 10^{16}$ and reasonable values of the unknown parameters e.g. $\alpha_i \approx 0.1$, $\gamma_i \approx 0.1$, $f_i \approx 1$, $v' \approx v_u$, we get $m_{0i} \simeq 20$ eV and $\epsilon \approx 1$ which leads us to the desired pattern of masses and mass differences where active neutrinos of each generation mix with the corresponding sterile neutrino maximally and the Δm^2 's scale as the cube of the corresponding charged fermion mass. Thus if we fix the $\Delta m_{ATMOS}^2 \sim 10^{-3} \text{ eV}^2$, then we get the vacuum oscillation solution for the solar neutrino puzzle.

This model also provides an explanation of the LSND results and the smallness of the mixing angle observed in the experiment is now similar to that observed in the quark sector and therefore easily understood by the same mechanisms that provide an explanation of the small quark mixing angles.

XI. MIRROR UNIVERSE MODEL OF THE STERILE NEUTRINO

The second suggestion that explains the ultralightness of the ν_s is the mirror matter model [48,47] where the basic idea is that there is a complete duplication of matter and forces in the universe (i.e. two sectors in the universe with matter and gauge forces identical prior to symmetry breaking). The mirror sector of the model will then have three light neutrinos which will not couple to the Z-boson and would not therefore have been seen at LEP. We denote the fields in the mirror sector by a prime over the standard model fields. We will call the ν'_i as the sterile neutrinos of which we now have three. The lightness of ν'_i is dictated by the mirror $B' - L'$ symmetry in a manner parallel to what happens in the standard model. Thus the ultralightness of the sterile neutrinos is understood in the most “painless” way.

The two “universes” are assumed to communicate only via gravity [86,87] or other forces that are equally weak. This leads to a mixing between the neutrinos of the two universes and can cause oscillations between ν_e of our universe to ν'_e of the parallel one in order to explain for example the solar neutrino deficit.

At an overall level, such a picture emerges quite naturally in superstring theories which lead to $E_8 \times E'_8$ gauge theories below the Planck scale with both E_8 s connected by gravity. For instance, one may assume the sub-Planck GUT group to be a subgroup of $E_8 \times E'_8$ in anticipation of possible future string embedding. One may also imagine the visible sector and the mirror sector as being in two different D-branes, which are then necessarily connected very weakly due to exchange of massive bulk Kaluza-Klein excitations.

In the mirror model, both sectors can either remain identical after symmetry breaking or there can be asymmetry. We will consider the second scenario. This was suggested in Ref. [48].

As suggested in Ref. [48], we will assume that the process of spontaneous symmetry breaking introduces asymmetry between the two universes e.g. the weak scale v'_{wk} in the mirror universe is larger than the weak scale $v_{wk} = 246$ GeV in our universe. The ratio of the two weak scales $\frac{v'_{wk}}{v_{wk}} \equiv \zeta$ is the only parameter that enters the fit to the solar neutrino data. It was shown in Ref. [48] that with $\zeta \simeq 20-30$, the gravitationally generated neutrino masses [56] can provide a resolution of the solar neutrino puzzle (i.e. one parameter generates both the required Δm^2_{e-s} and the mixing angle $\sin^2 2\theta_{e-s} \simeq 10^{-2}$). There can also be a large angle MSW fit with reasonable choice of parameters e.g. smaller ζ but with coefficients of higher dimensional operators allowed to vary between 0.3 to 3.

There are other ways to connect the visible sector with the mirror sector using for instance a bilinear term involving the righthanded neutrinos from the mirror and the visible sector. An $SO(10) \times SO(10)$ realization of this idea was studied in detail in a recent paper [88], where a complete realistic model for known particles and forces including a fit to the fermion masses and mixings was worked out and the resulting predictions for the masses and mixings for the normal and mirror neutrinos were presented. In this model, the fermions of each generation are assigned to the $(\mathbf{16}, \mathbf{1}) \oplus (\mathbf{1}, \mathbf{16}')$ representation of the gauge group. The $SO(10)$ symmetry is broken down to the left-right symmetric model by the combination of $\mathbf{45} \oplus \mathbf{54}$ representations in each sector. The $SU(2)_R \times U(1)_{B-L}$ gauge symmetry in turn is broken by the $\mathbf{126} \oplus \mathbf{126}$ representations. These latter fields serve two purposes: first, they guarantee automatic R-parity conservation and second, they lead to the see-saw suppression for the neutrino masses. We do not get into detailed discussion of the masses and mixings of neutrinos in such a model here. The mixing between the two sectors is caused by a multiplet belonging to the representation $(\mathbf{16}, \mathbf{16}') + (\mathbf{16}, \mathbf{16}')$. If the mass of this last multiplet is kept at the GUT scale, the mixing between the two righthanded neutrinos caused by this is large and the ensuing effects on the light neutrino mixings are small.

Since the mirror matter model has many ultralight particles, the issue of consistency with big bang nucleosynthesis must be addressed. Recall that present observations of Helium and deuterium abundance can allow for at most 4.53 neutrino species [35] if the baryon to photon ratio is chosen appropriately. Since the model has three extra neutrinos, an extra photon and the extra e^+e^- pair, apriori, the effective neutrino count in the model could be as large as 8.2. However, in the asymmetric mirror model, since the neutrinos decouple above 200 MeV or so due to weakness of the mirror Fermi coupling, their contribution at the time of nucleosynthesis is negligible (i.e. they contribute about 0.3 to δN_ν .) On the other hand the mirror photon could be completely in equilibrium at $T = 1$ MeV so that it will contribute $\delta N_\nu = 1.11$. There is also a contribution from the mirror electron-positron pair of $N_\nu = 2$. All together the total contribution to N_ν is less than 6.4. So to solve this problem, we invoke an idea called asymmetric inflation whereby the reheating temperature in the mirror sector is lower than the reheating temperature in the familiar sector. Models with this kind of possibility was discussed in [89].

There may be another potentially very interesting application of the idea of the mirror universe. It appears that there may be a crisis in understanding the microlensing observations [90]. It has to do with the fact that the best fit mass for the 14 observed microlensing

events by the MACHO and the EROS group is around $0.5M_{\odot}$ and it appears difficult to use normal baryonic objects of similar mass such as white dwarfs to explain these events, since they lead to a number of cosmological and astrophysical problems [91]. Speculations have been advanced that this crisis may also be resolved by the postulate that the MACHOs with $0.5M_{\odot}$ may be mirror stars [92,93] which would then have none of the difficulties that arise from interpretations in terms of conventional baryonic matter. In this model, mirror baryons can form the dark matter of the universe so that there is no need for a neutralino dark matter.

XII. CONCLUSIONS AND OUTLOOK

In this review, we have tried to provide a brief look at the new physics implied by the discovery of neutrino oscillations. We start with a summary of the possible textures for neutrino masses implied by the present data with and without the inclusion of the LSND data. Several three, four and six neutrino scenarios have been outlined. We then briefly describe the various ideas primarily aimed at understanding the smallness of the neutrino masses both in the context of grand or simple unified models. Finally two theories that can naturally incorporate an ultralight sterile neutrino are discussed. Clearly, there is considerable subjective judgement used in the selection of models and an apology is due to all authors whose models are not discussed here. Limitations of space is certainly one simple excuse that can be given. In any case, the final story in this field is yet to be written and it could very easily be that the models described here do not eventually win out. Even that will be a very valuable piece of information since the ideas touched on here are certainly some of the more salient ones around now. But it is the fervent hope of this author (presumably shared by many workers in the field) that some of the ideas described in the literature (and reviewed here) will survive the test of time. The “ball” is right now in the court of the experimentalists and we pin our hopes on the large number of continuing (SNO, Borexino, K2K, MINOS) as well as planned (KAMLAND, ICARUS, LENS, GENIUS, CUORE, ORLAND etc) experiments in this field. On the theoretical side, true progress can be said to have been made only in understanding the smallness of the neutrino masses in different scenarios but the complete picture of mixing angles is far from being at hand, although there are many models that under various assumptions lead to realistic fits. One of the hardest theoretical problems is to understand the ultrasmall mass difference that would be required if the vacuum oscillation solution to the solar neutrino problem eventually wins the race (although there exist a handful of interesting suggestions [49,94,47]). One will then have to check whether the radiative corrections in these models destabilize the result.

The bottom line is that the field of neutrino mass has become one of the most vibrant and exciting fields in particle physics as we move into the new millenium and is at the moment the only beacon of new physics beyond the standard model.

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FIGURES

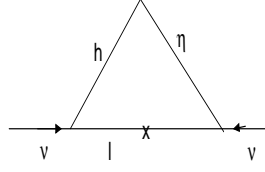


FIG. 1. One loop diagram for neutrino mass in the Zee model

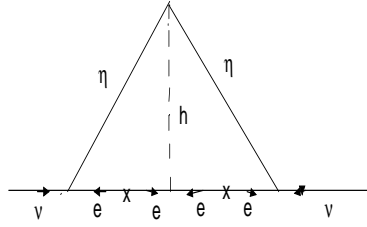


FIG. 2. Two loop diagram for neutrino mass in the Babu model